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STATISTICAL PROCESS CONTROL: NEW CHARTING TECHNIQUE

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ABSTRACT

The work presents the new charting technique for complete process analysis from both the stability and capability points of view. Stability evaluation is performed using the true control region on a two-dimensional graph of the measures of the central tendency and spread of the process. The region boundary represents the locus of points with the same probability density function value and is characterized by a one-dimensional statistic, depending on the standardized sample average and standard deviation. The chronological record of the statistic can be used for testing the statistical behaviour of the process, a procedure characterized by reduced adjustment errors compared with the conventional Shewhart charts. Capability assessment is based on time-sequence analysis of the quality loss point estimator, which value depends on the sample variance and the squared distance from the $\bar{x}$-chart centerline to the sample average. The proposed approach allows to apply the charting technique for simultaneous analysis of both the statistical and quality behaviour of the process and, thereby, optimize process control.

INTRODUCTION

Statistical Process Control represents a set of procedures intended mainly for stability and capability evaluation. The former reflects the statistical behaviour of the process, whereas the latter characterizes its ability to produce the product with the given property, in other words - its quality level.

The conventional procedure of process statistical behaviour analysis is based on control charts [1]. Variables control charts, representing one of the main tools in Quality Control, consist in separate testing of hypotheses about the parameters of the process central tendency and spread. The positive decision whether a sampling point belongs to the population is based on the logical 'and' concerning these hypotheses: 'the former and the latter are true'.

Process central tendency is usually controlled with a control chart for averages ($\bar{x}$-chart), whereas process variability can be controlled with either a control chart for the standard deviation (s-chart) or a control chart for the range (R-chart). In this work we will concentrate on the combination of $\bar{x}$- and s-charts, although the inferences concerning the conventional technique and the proposed approach are valid for $\bar{x}$-R charts as well. The use of separate $\bar{x}$- and s-charts is equivalent to plotting ($\bar{x}$, s) on a two dimensional graph formed by superimposing the $\bar{x}$-chart over the s-chart, as shown in Fig. 1. The upper and lower control limits (UCL and LCL, respectively) plotted on the corresponding axes form a rectangle (referred to as the Shewhart Rectangle) representing the control region for this $\bar{x}$-s graph. For all points falling within the Rectangle the process is considered to be 'in control', otherwise it is in the 'out-of-control' (OOC) state in terms of the mean (zones 2, 7), of the variance (zones 4, 5) or of both of them (zones 1, 3, 6, and 8).

Unlike the Shewhart Rectangle, the true control region has an elliptical shape and represents the result of cutting the $\bar{x}$-s joint sampling distribution by a horizontal plane at the height corresponding to the given significance level. The new technique for stability testing, presented in the first section of the work, is based on the chronological record of the one-dimensional statistic, depending on standardized sample average and standard deviation, and its comparison with critical value, corresponding to the boundary of the true control region.

In contrast to the continuous procedure of charting, the capability assessment performed by periodical auditing, is confined to evaluating the past and forecasting the future. It does not involve current capability monitoring. Conventional capability study yields means (in the form of dimensionless indices) for measuring the process variability and comparing it against the specification limits [2], while modern capability assessment is based on the Taguchi target oriented loss function [3]. Both these approaches yield relevant benchmarks, but 'a poor basis for continuous process control and improvement' [4].

A technique of process quality level monitoring is developed in the second section. For current quality evaluation we propose to use the chronological record of the quality loss point estimator representing the second moment about the $\bar{x}$-chart centerline. The last section of the work is devoted to a procedure of complete process analysis using combination of the proposed charts.

Average($\bar{x}$)	UCL(s)	LCL(s)	
Zone 1 OOC state due to excessive variation of both population mean and variance	Zone 2 OOC state due to excessive variation of population mean	Zone 3 OOC state due to excessive variation of both population mean and variance	UCL($\bar{x}$)
Zone 4 OOC state due to excessive variation of population variance	CONTROL REGION (Shewhart Rectangle)	Zone 5 OOC state due to excessive variation of population variance	LCL($\bar{x}$)
Zone 6 OOC state due to excessive variation of both population mean and variance	Zone 7 OOC state due to excessive variation of population mean	Zone 8 OOC state due to excessive variation of both population mean and variance	
		Standard deviation (s)	

Fig. 1. Graphical description of the variables control charts decision making method

PROCESS STABILITY EVALUATION

On the assumption that samples of size n are drawn from a normal population with parameters μ and σ, the probability density function (PDF) of the $\bar{x}$-s joint sampling distribution can be written as follows [5]

$$p(\bar{z}, s^*) = \sqrt{\frac{2n}{\pi}}\,\frac{e^{-\frac{n-2}{2}}}{\Gamma\left(\frac{n-1}{2}\right)}\left(\frac{n-2}{2}\right)^{\frac{n-1}{2}} \exp\left(-\frac{n\bar{z}^2-(n-2)\left[\ln(s^*)^2-(s^*)^2-1\right]}{2}\right) \quad (1)$$

Where $\bar{z} = \frac{\bar{x}-\mu}{\sigma}$ and $s^* = \frac{s}{\sigma}\sqrt{\frac{n-1}{n-2}}$

The general assumption lying in the basis of the suggested approach is that the true control region boundary represents the locus of points with the same PDF value. The iso-PDF contour is described by the exponent power in braces, which can be designated as the B-statistic

$$B = 0.5(n\bar{z}^2 - (n-2)[\ln(s^*)^2 - (s^*)^2 + 1]) \qquad (2)$$

It can be shown [6] that the B-statistic is exponentially distributed with zero minimal value and unity expectation and variance. The probability of finding a sample point $(\bar{x}, s)$ inside the control iso-PDF contour is given by

$$\int_0^{B_{crit}} p(B)\, dB = 1 - \alpha \qquad (3)$$

where $B_{crit} = \ln(\alpha)$ is the critical B-value depending on the accepted significance level (α). Since it is customary to use $\alpha = .0027$ for conventional control charts, one can get for the joint distribution: $\alpha = 1-(1-.0027)^2 = .0054$.
The difference between the Shewhart Rectangle, constructed with the aid of the corresponding control chart factors, and the proposed elliptical region is illustrated by simulation of a process with normal random variation N(0,1). The results of sampling (set of 1800 samples, n=5) are shown in Fig. 2. One can see that the oval computed according to $\alpha = .0054$ suits better the shape of the scatter diagram. None of the points falls in the Rectangle corners (underadjustment), whereas there are points falling inside the oval and outside the Rectangle (overadjustment), although the oval area is approximately 10% less than that of the Rectangle. It can be shown [6] that the elliptical control region, bounded by the iso-PDF contour, represents the unbiased most powerful test.
The chronological record of the statistic represents a B-chart with single UCL (B_{crit}): for all pairs $(\bar{x}_i, s_i)$ falling within the control region or on its boundary $B_i \leq B_{crit}$, whereas for all pairs $(\bar{x}_j, s_j)$ falling outside the region $B_{crit} < B_j$. The proposed B-chart can be used for process state testing characterized by reduced adjustment errors compared with the conventional control charts.

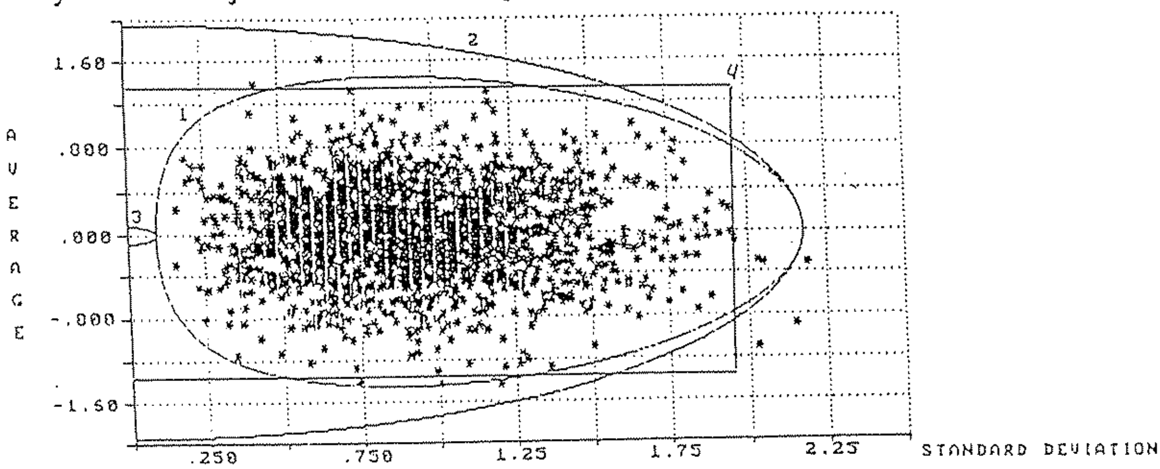

Fig. 2. x̄-s graph presenting the results of Monte-Carlo simulation: 1- control oval, 2- larger iso-loss semiellipse, 3- smaller iso-loss semiellipse, 4- Shewhart Rectangle, *- sampling data

LOSS ESTIMATE CHART

Stability is a necessary but insufficient property of a high-technology process which should be also capable to yield products with the same (or similar) desired values of the controlled quality characteristics. This ability (so-called process uniformity) implies a loss in quality when the process deviates from its expected value and generates non-uniform products. For current uniformity evaluation we propose to use a loss point estimator (LE) associated with the second moment about the $\bar{x}$-chart centerline, which depends on the sample variance and the squared distance from the centerline to the sample average $\bar{x}_i$. For standardized sample statistics the expression for the LE-statistic should be written as follows

$$LE = \frac{(n-1)}{n}(s^*)^2 + \bar{z}^2 \quad (4)$$

Note that for a properly centered process the $\bar{x}$-chart centerline coincides with the specification target. The chronological record of this statistic can be used for continuous process quality monitoring. Obviously, the idea of the LE-chart is based on the Taguchi philosophy, and the LE-statistic closely resembles the Taguchi Expected Loss (EL) characterizing the quality of a normally distributed product in a symmetrical bilateral tolerance [3].

The process can be considered as uniform if its quality loss does not exceed the extreme loss levels under random statistical behaviour, hence the lower and upper loss can be obtained by comparison of the LE-statistic and values corresponding to the extreme coordinates of the control region boundary, which in turn are obtainable by substitution of the expression for B_{crit} in Eq. (2). Analysis of the extreme loss levels when the process behaves in a random fashion shows that the 'loss control region' on the $\bar{x}$-s graph represents a 'quality loss annulus' bounded by two concentric iso-loss semiellipses. Their common center coincides with the coordinate origin and the major semiaxes are bounded by the leftmost and rightmost coordinates (on the s-axis) of the contour bounding the control region. The smaller and lager semiellipses are loci of points characterized by the lower (LCL_{LE}) and upper (UCL_{LE}) quality iso-loss level, respectively. An 'annulus' calculated for the results of Monte-Carlo simulation is shown in Fig. 2.

It is obviously unprofitable to adjust a process whose loss does not exceed the extreme limits under random statistical behaviour. Thus the probability of overadjustment, unnecessary from the quality point of view, can be reduced thanks to the fact that for any point within the 'loss annulus' (even if it falls outside the control region) the LE-statistic does not exceed its control limits.

COMPLETE PROCESS ANALYSIS USING THE B- AND LE-CHARTS

The area under the $\bar{x}$-s graph is subdivided by the control region and the 'quality loss annulus' into three distinct zones: 1-st zone - within the region; 2-nd zone - between the region boundaries and the iso-loss semiellipses; 3-rd zone - outside the annulus. This subdivision is applicable in complete process analysis: 1-st zone - the process is both stable and uniform from the quality point of view: both the B- and LE-chart are 'in control'; 2-nd zone - the process is unstable but uniform: the B-chart is 'out of control', whereas the LE-chart is 'in control', i.e. the process is uniform today, but may not be tomorrow; 3-rd zone - the process is both unstable and non-uniform: both the B- and LE-chart are 'out of control'. Thus at the start-up phase of a new process the B-chart is used, it must be 'in control' before the conventional capability study (or assessment according to the Taguchi approach) is performed. Under routine operation both the B- and LE-chart are used: the former serves for warning, whereas the latter is of decisive importance, because what counts for the customers is not so much whether the process is stable, but whether its quality is stable.

CASE STUDY

The viscosity of the 25 samples of six was measured. The samples were used to construct the $\bar{\bar{x}}$-, s-, B- and LE-charts shown in Figs. 3a, 3b, 3c and 3d, respectively. The comparative analysis of the conventional and proposed charts shows that process management according to the former leads both to overadjustment (point 14 - from both the stability and uniformity points of view; points 6 and 12 - from the quality point of view only), and to underadjustment (point 9), when no appreciable change due to an assignable cause (simultaneous shift of the process mean and increase of the process variability) had been detected.

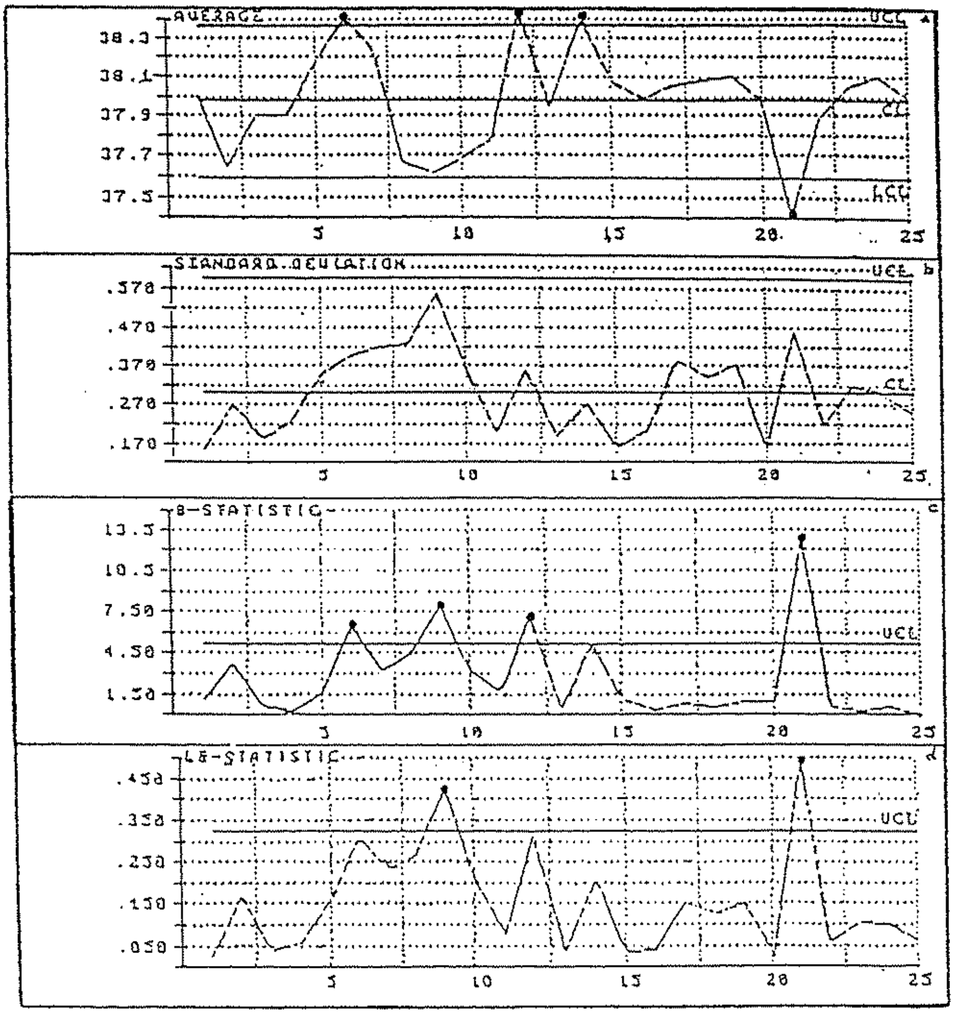

Fig. 3 Control charts for averages (a), standard deviations (b), B-statistic (c) and LE-statistic (d)

CONCLUSION

Process stability testing by means of conventional control charts for variables is equivalent to construction of a rectangular control region, which cannot be considered as optimal, so that reliance on two separate 'uniparameter' Shewhart charts can be misleading. The proposed alternative technique is based on a B-chart representing the chronological record of the statistic,

describing elliptical (oval) iso-PDF contours of the joint $\bar{x}$-s sampling distribution. The single UCL on the chart corresponds to the control region boundary on the two-dimensional $\bar{x}$-s graph. The process state testing by means of the proposed B-chart, leads to reduction of the adjustment errors compared with the conventional Shewart charts.
Another suggested technique treats the problem of process analysis from the quality point of view. The LE-statistic, represented by the second moment about $\bar{\bar{x}}$, is used for continuous evaluation of process uniformity. The control limits for the LE-chart correspond to iso-loss semiellipses, characterizing the extreme losses occurring while the random statistical behaviour of the process. At the start-up phase of a new process the B-chart is used for stability testing with the attendant reduction of adjustment errors compared with the conventional charts. Under routine operation simultaneous evaluation of both process stability and uniformity is performed with the suggested B- and LE-charts used together. This allows to reduce the probability of overadjustment, unnecessary from the quality point of view.

APPENDIX

The proposed one-dimensional statistics form a reliable basis for assessing the process characteristics, but cannot distinguish between the different assignable causes of process disturbance (shift of the mean or a variation of the variance). To remedy this shortcoming an alternative charting technique was developed in the framework of the suggested approach. The technique is based on setting up the $\bar{x}$-chart with double variable control limits ('pooled' chart). The charting procedure implies the continuous revision of the limits depending on the sample standard deviation value. The inner couple of limits, computed using the equation of the control region boundary, are used as the action limits at the start-up phase and as warning limits at the routine operation. The outer couple of limits are calculated in accordance with the equations of the iso-loss semiellipses, outlining 'quality loss annulus', and replace the action limits in the later process stage. The 'pooled' chart characterizes the process as a whole: it contains the information concerning both the process central tendency ($\bar{x}$-plot) and spread (control limits). A detailed description of the chart could not be presented in the paper due to limitations of space.

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